The Complexity of Object Oriented Type Analysis

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The Problem

- **Given:** an OO (class hierarchy) program
- **Find:** the set of dynamic types that each polymorphic variable may contain at any given program point
- **Example:**

```
struct B { virtual void f(); } b, *p;
struct D1: B { void f(); } d1;
struct D2: B { void f(); } d2;

void B::f()  { p = d1; p->f(); }
void D1::f() { p = d2; p->f(); }
void D2::f() { p = d1; }

int main()
{
    p = random() > 0.5 ? d1 : b;
    p->f();

    // What is the dynamic type(s) of p here?
}
```

Motivation

- **Polymorphic calls are expensive:**
  - **Dynamic dispatch mechanism:** determine the dynamic type of an object, then call the right method
  - **Lost optimization opportunities:** difficult to inline call
    - Many methods are very small
- **Polymorphic calls are rare:**
  - **A large portion of call sites are monomorphic**
  - **A large portion of actual calls are monomorphic**
- **Static analysis (state of the art):**
  - **Complex algorithms**
  - **Time consuming**
  - **Inaccurate**
  - **Heuristics, difficult to compare**

An Undecidable Problem?

```
void g()
{
    if (main() == 0)
        p = new D1;
    else
        p = new D2;

    // What is the dynamic type(s) of p here?
}
```

The fundamental hypothesis of static analysis

All paths in the control-flow graph are probable

Dimensions of Static Analysis

- **Control flow:**
  - **Flow insensitive:** ignore control flow information. Statements may be executed in an arbitrary order and arbitrarily many times
    - Used e.g., in standard and non-standard type inference
  - **Flow sensitive:** analysis may not infer the truth value of conditions (e.g., in **if** and **loop** statements)
  - **Object Oriented:** analysis may deduce (and use) the actual method called
  - **Heuristic:** analysis infer (and use) the truth vale of conditions
- **Data types:** simple / multi-level pointer/ recursive
- **Program structure:** intra-procedural / non-recursive / recursive

Challenge: determine the complexity of every combination of the parameters

Multi-Level Pointers

```
template <unsigned n>
    class S {
        public:
            S<n-1> *left, *right;
    };

#define N 1000

class S<N> a;
```

Reminder: Complexity Classes

- **Time complexity classes:**
  - **P, NP, EXP-TIME, DEXP-TIME, ...**
- **Space complexity classes:**
  - **P-SPACE, EXP-SPACE, DEXP-SPACE, …**
- **Relationship:**

$$\mathrm{P} \subseteq \mathrm{NP} \subseteq \mathrm{P\text{-}SPACE} \subseteq \mathrm{EXP\text{-}TIME} \subset \mathrm{EXP\text{-}SPACE} \subseteq \mathrm{EXP\text{-}TIME}$$

- **Problem P is Complexity Class *<u>X-hard</u>*:**
  - **P is more difficult than <u>all</u> problems in X.**
- **Problem P is Complexity Class *<u>X-complete</u>*:**
  - **P is X-hard**
  - **P is in X**

Previous Results

- **Pande and Ryder: type analysis is NP-Hard**
  - **object oriented analysis**
  - **non-recursive data structures**
  - **non-recursive programs**
    - (intra-procedural is not interesting in OO analysis)
- **Hurwitz: alias-analysis (may point-to) is NP-hard**
  - **flow insensitive**
  - **recursive data-structures**
  - **non-recursive programs**

Our Results

Recursion	Multi-level pointers	Recursive Data Types	Complexity
No	No	No	P-SPACE complete
Yes	No	No	EXP-TIME complete
No	Yes	No	EXP-SPACE complete
Yes	Yes	No	DEXP-TIME complete
—	—	Yes	Undecidable

P-Space Hard

- **Basic idea**: reduce the halting problem of a linear bounded automaton.
- **LBA:** a Turing machine whose tape length is bounded by a linear function of the input size.
- **Ingredients of a Turing machine:**
  - **Q:** a finite set of internal states
  - $\mathbf{q_0} \in$ **Q:** an initial state
  - $\mathbf{f} \in$ **Q;** an accepting state
  - **T:** a working tape of length n of {0,1} symbols
  - δ: $\mathbf{Q} \times \{0,1\} \rightarrow \mathbf{Q} \times \{0,1\} \times \{\text{prev, next}\}$: a transition function, specifying for each internal state and current tape head value:
    - next state
    - value to write on the tape
    - direction of head movement

Encoding Techniques

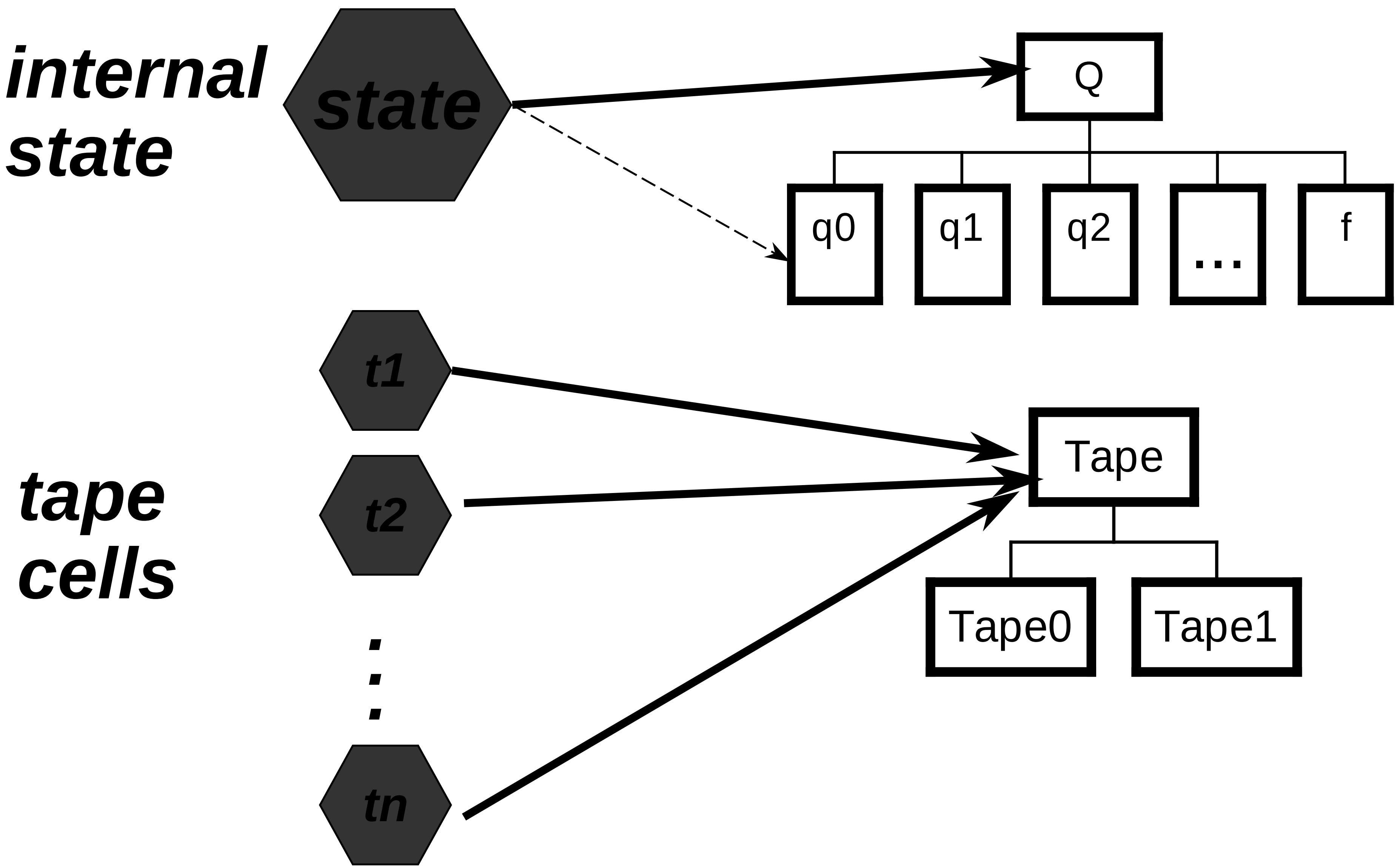
- **Type encoding:**
  - **Value:** Each distinct value in a Turing machine is encoded as a distinct type.
  - **Variable:** A variable of a Turing machine which may assume different values, is encoded as a polymorphic program variable which may assume different dynamic types
  - **Examples:**
    - Set of internal states / current LBA state
    - {0,1} set of tape cell values / each tape cell
- **Dynamic binding encoding:** a different implementation of a virtual function for each different type representing value
- **Multiple dispatch:** to realize a function depending on several values
- **Main difficulty:** encode tape head position

Encoding LBA’s Current State

```
class Q {
    public:
        virtual void delta0() = 0;
        virtual void delta1() = 0;
};
```

```
class Tape {
    public:
        virtual void delta() = 0;
};
class Tape0: public Tape {
    public:
        virtual void delta() { state->delta0{} }
};
class Tape1: public Tape {
    public:
        virtual void delta() { state->delta1(); }
};
```

Class Object Diagram



Initialization and Termination

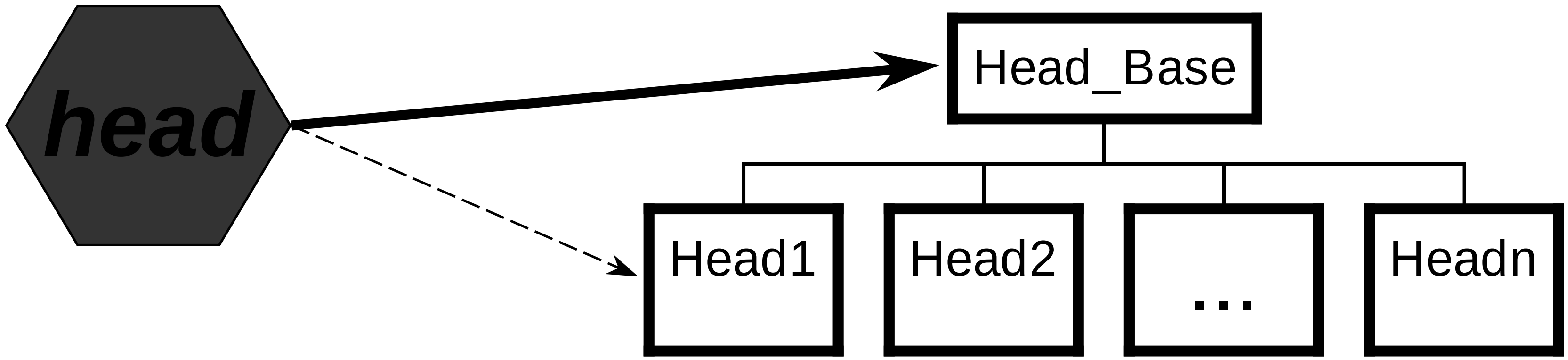
```
Q *state = new Q0;

Tape *t1 = new Tape0;
Tape *t2 = new Tape1;
…
Tape *tn = new Tape0;

main()
{
    … // Turing machine emulation goes here

    // Is F a dynamic type of state?
}
```

Encoding Tape Head Position




```
class Head_base {
   public:
      virtual void next() = 0;
      virtual void prev() = 0;
      virtual void set0() = 0;
      virtual void set1() = 0;
      virtual void delta() = 0;
} *head;
```

Using a Template for Generating Classes for Head Positioning

```
template <int i>
class Head: public Head_base {
public:
    static Tape *t;
    void next() { head = new Head<i+1>; }
    void prev() { head = new Head<i-1>; }
    void set0() {
      Head<i>::t = new Tape0;
    }
    void set1() {
      Head<i>::t = new Tape1;
    }
    void delta() {
      Head<i>::t -> delta();
    }
};
```

Template specialization for boundary cases not shown

Putting it all Together: the Delta function

- **Consider the following polymorphic call**

```
head->delta()
```

- **Suppose that** `head` **is of type** `Head<17>` (head is on position 17 on tape)

```
void Head<17>::delta() {
   Head<17>::t->delta(); }
```

- **Suppose that** `Head<17>::t` **is of type** `Tape1` (cell 17 on the tape has 1)

```
void Tape1::delta() {
   Head<17>::state->delta1(); }
```

- **Suppose that** `state` **is of type** `Q4` (machine is in internal state 4)

...

The Delta Function (cont.)

- **Suppose that** `state` **is of type** `Q4` (machine is in internal state 4)

```
void Q4::delta0() {
  state = new Q7; // State transition
  head->set0();
  head->next();
}
```

- **Writing to tape:**

```
void Head<17>::set0() {
  Head<17>::t = new Tape0;
}
```

- **Moving the head**

```
void Head<17>::next() {
  head = new Head<18>;
}
```

The Main Function

```
main()
{
    head = new Head<0>;

    for (;;) {
      head -> delta();
    }
    //  What is the type of state here?
}
```

- **No recursion used**
- **No recursive data structures**
- **As flow-insensitive as OO analysis can be**
- **O(n) classes**
- **O(1) Inheritance depth**

P-Space Completeness

- **Assume:**
  - **No recursive data types**
  - **No recursive calls**
  - **All variables are global**
- **Reduction to:** s-t connetivity in exponentially sized graph
  - **Nodes:** a tuple of (1) program counter location, (2) function call stack, and (3) an assignment of types to all program variables
    - Each node is described by a vector of polynomial size
    - Total number of nodes is exponential
  - **Statements:**
    - **Simple:** a simple edge in the graph
    - **If statement:** two edges, each corresponding to a branch
    - **Loop condition:** same as “if” statement
    - **Loop body:** a loop edge
    - **Function call:** an edge

Dealing with Recursion

- **Main Difficulty:** memory is unbounded...
- **Auxiliary push-down machine (aux-PDM):** Turing machine + Unbounded stack.
- **Theorem (Cook)** ***:*** Let *w(n)* >= *lg(n)*. Let *A* be a language. Then, the following are equivalent:
  - **A is accepted by some <u>deterministic</u> aux-PDM within storage *w(n)*.**
  - **A is accepted by some <u>non-deterministic</u> aux-PDM within storage *w(n)*.**
  - **A is accepted by some time-bounded computer within time $2^{c\, w(n)}$, for some constant c > 0.**

Other Complexity Results

- **EXP-TIME:** ***Reduction to an aux-PDA (Turing machine with stack)***
- **EXP-SPACE:** *With multi-level pointers, space might be exponential*
  - **simulate a turing machine with exponential size tape.**
- **DEXP-TIME:** Same as EXP-TIME, but with exponential sized tape.
- **UNDECIDABLE:** Space is unbounded, hence can simulate a general turing machine.

Scope Size

- **Scope size of a routine:** *total number of accessible variables*
  - **Parameters**
  - **Local variables**
  - **Global variables**
  - **Instance variables (defined and inherited)**
- **Scope size of a program:** ***maximal SS over all routines***
- ***Theorem:*** if the SS of a program p of size n is at most log n, then, OOTA of p can be done in sub-exponential time: $n^{O(\log(n))}$
  - **Recursion allowed**
  - **No recursive data types**
  - **Multi-level pointers to the extent allowed by SS <= log n**

Chameleonicity

- **A class C:** *Chm(C) := Rnd(lg(N) + 0.5).*
  - **N= Number of concrete classes that inherit from C, including C itself.**
  - **Chm(C)= # bits required to represent the run time type a variable of type C.**
- **A varaible V of type C:** *Chm(V) := Chm(V)*
- **A routine R:** *Chm(R) := Sum(Chm(V), V in R)*
  - **Chm(p) = # bits required to represent the run time type of all variables in R**
- **A prograp P:** Chm(P) := Max(Chm(R), R in P)
  - **Hypothesis:** ***for "well written programs", chm(P) = O(lg n)***
- **Theorem:** the time for solving OOTA of P is

$$2^{c*chm(P)}$$

Open Problems

- Exact characterization of the may-alias, flow-insensitive analysis problem
- Study of the flow-insensitive model of computation.

- Barth J.
- CACM vol 21 # 9 , page 724-736
- A practical inter procedural data flow analysis

Other Results

- Basic: No-recursion/Simple data types
  - P-Space Complete
- + Recursion:
  - EXP-TIME complete
- Basic + Multi-level pointers
  - EXP-SPACE complete
- Basic + Multi-level pointers + recursion
  - DOUBLE-EXP-TIME COMPLETE
- Recursive data structures
  - Undecidable!
- No global variables, and each function has only a constant number of local variables, no arrays. Each type has a fixed number of slots?
  - Equivalence of two programs is decidable.
  - Langmaack, 72/3 acta informatica.
  - Passing parametes to programs.
  - If each type is finite then the halting program is decidable.

G

- **Golubski, Wolfgang, PHD: Smalltalk programs are decidable method is formal reachable.**
  - **Control-flow sensitive**
  - **Inter-procedural analysis**
- **Reference.**